Stochastic Differential Equation for a System Coupled to a Thermostatic Bath via an Arbitrary Interaction Hamiltonian

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The conventional Langevin equation offers a mathematically convenient framework for investigating open stochastic systems interacting with their environment or a bath. However, it is not suitable for a wide variety of systems whose dynamics rely on the nature of the environmental interaction, as the equation does not incorporate any specific information regarding that interaction. Here, we present a universal formulation of the stochastic differential equation (SDE) for an open system coupled to a thermostatic bath via an arbitrary interaction Hamiltonian. This SDE encodes the interaction information to a fictitious potential (mean force) and a position-dependent damping coefficient. Surprisingly, we find that the conventional Langevin equation can be recovered in the presence of arbitrary strong interactions given two conditions: translational invariance of the potential and disjoint separability of the bath particles. Our results provide a universal framework for studying open stochastic systems with an arbitrary interaction Hamiltonian and yield deeper insight into why various experiments fit the conventional Langevin description regardless of the strength or type of interaction.