Pair density waves and exotic superconductivity have attracted much recent attention. We present a joint theoretical and experimental study of fermion pairing with finite-momentum, or the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) order. Experimentally a doped and spin-imbalanced attractive Hubbard model is realized with cold atomic gas in an optical lattice, and quantum gas microscopy is used to probe its properties. Computationally we study the same model with state-of-the-art constrained-path (CP) auxiliary field quantum Monte Carlo (AFQMC). Direct comparison between them on various short-range magnetic and charge correlations versus temperature shows excellent agreement. We then systematically investigate the pairing correlations with CP AFQMC as temperature is lowered, and determine parameter regimes in density and magnetization where FFLO order is observed. We show that the temperature at which such order appears is already within reach of current experiment. We discuss routes for experimental detection of FFLO correlations.