Lieb-Schultz-Mattis Theorem in Open Quantum Systems

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The Lieb-Schultz-Mattis (LSM) theorem provides a general constraint on quantum many-body systems and plays a significant role in the Haldane gap phenomena and topological phases of matter. Here, we extend the LSM theorem to open quantum systems and establish a general theorem that restricts the steady state and spectral gap of Liouvillians based solely on symmetry. Specifically, we demonstrate that the unique gapped steady state is prohibited when translation invariance and U(1) symmetry are simultaneously present for noninteger filling numbers. As an illustrative example, we find that no dissipative gap is open in the spin-1/2 dissipative Heisenberg model, while a dissipative gap can be open in the spin-1 counterpart—an analog of the Haldane gap phenomena in open quantum systems. Furthermore, we show that the LSM constraint manifests itself in a quantum anomaly of the dissipative form factor of Liouvillians. We also find the LSM constraints due to symmetry intrinsic to open quantum systems, such as Kubo-Martin-Schwinger symmetry. Our work leads to a unified understanding of phases and phenomena in open quantum systems.

Reference: K. Kawabata, R. Sohal, and S. Ryu, Phys. Rev. Lett. **132**, 070402 (2024).