

# Spontaneous symmetry breaking in open quantum systems: strong, weak, and strong-to-weak

Ding Gu, Zijian Wang, and Zhong Wang\*

Institute for Advanced Study, Tsinghua University, Beijing, 100084, China

\*Email: wangzhongemail@tsinghua.edu.cn

**Abstract:** Depending on the coupling to the environment, symmetries of open quantum systems manifest in two distinct forms, the strong and the weak. We study the spontaneous symmetry breaking among phases with strong symmetry, weak symmetry, and no symmetry. Concrete Liouvillian models with strong and weak symmetry are constructed, and different scenarios of symmetry-breaking transitions are investigated from complementary approaches. It is demonstrated that strong symmetry always spontaneously breaks, either completely, or into the corresponding weak symmetry. For strong  $U(1)$  symmetry, we show that strong-to-weak symmetry breaking leads to gapless Goldstone modes dictating diffusion of the symmetry charge in translational invariant systems. We conjecture that this relation among strong-to-weak symmetry breaking, gapless modes, and symmetry-charge diffusion is general for continuous symmetries. It can be interpreted as an "enhanced Lieb-Schultz-Mattis (LSM) theorem" for open quantum systems, according to which the gapless spectrum does not require non-integer filling. We also investigate the scenario where the strong symmetry breaks completely. In the symmetry-broken phase, we identify an effective Keldysh action with two Goldstone modes, describing fluctuations of the order parameter and diffusive hydrodynamics of the symmetry charge, respectively. For a particular model studied here, we uncover a transition from a symmetric phase with a "Bose surface" to a symmetry-broken phase with long-range order induced by tuning the filling. It is also shown that, in both weak and strong symmetry cases, the long-range order of  $U(1)$  symmetry breaking is possible in spatial dimension  $d \geq 3$ . Our work outlines the typical scenarios of spontaneous symmetry breaking in open quantum systems, puts forward a theoretical framework to characterize them, and highlights their physical consequences.

## Reference

Ding Gu, Zijian Wang, Zhong Wang, [arXiv:2406.19381](https://arxiv.org/abs/2406.19381)