2D moiré materials, such as the twisted bilayer graphene (TBG) and twisted transition metal dichalcogenides, have emerged as exciting physical platforms featuring isolated narrow bands with non-trivial topology and strong electronic interactions. Unlike traditional solid state materials, the large moiré unit cell has enabled the study of interaction-induced phenomena in the Hofstadter regime, when an externally applied magnetic flux per unit cell is comparable to the magnetic flux quantum hc/e. Despite their significance, the Chern insulating states observed at a finite magnetic field – with Chern number t and extrapolating to a band filling s at zero field – remain poorly understood. Unraveling their nature is among the most important open problems in the province of moiré materials.