Emergence of topological constraint in kinetically constrained spin model

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Kinetically constrained spin models are toy models of supercooled liquids and amorphous solids. In this presentation, I discuss the prototypical Fredrickson–Andersen (FA) kinetically constrained model from the viewpoint of K-core combinatorial optimization. In the FA model, each vertex in a graph has two states (empty or being occupied), and a vertex is temporarily unflippable (blocked) if it has K or more occupied nearest neighbors.

I prove that each kinetic cluster of the FA system, which contains all the mutually visitable microscopic occupation configurations following the FA local kinetic rules, is exactly the solution space of a specific instance of the K-core attack problem. Especially, the fully unfrozen kinetic cluster, which contains all the occupation configurations visitable starting from the all-empty one and within which every vertex is flippable in the long-time limit, is completely specified by a single topological constraint: the occupied vertices should not sustain a K-core (a K-core is a subgraph within which each vertex is connected to at least K other vertices).

The possible thermodynamic phase transitions of the FA kinetic model can therefore be rigorously studied by considering the K-core attack problem. The spin glass phases in the K-core attack problem are induced by the single global constraint of K-core-absence. I demonstrate some theoretical and numerical results obtained on the ensemble of random graphs. I also briefly mention a numerical simulation strategy for detecting the possible low-temperature spin glass phases in single graph instances.

Reference: Hai-Jun Zhou, “K-core attack, equilibrium K-core, and kinetically constrained spin system”, Chinese Physics B 33, 066402 (2024); arXiv:2404.16237 [cond-mat.dis-nn]