The interplay between topology and correlation has been a driving force in the study of moiré superlattices, both experimentally and theoretically. Recently, experimental observations of fractional topological phases in twisted bilayer $MoTe_2$ have garnered significant interest in this system. In this talk, I will present our recent work on correlated topological phases in twisted $MoTe_2$. Specifically, I will cover (a) a brief introduction to twisted bilayer $MoTe_2$, (b) an accurate modeling of twisted $MoTe_2$ at various twist angles, and (c) many-body calculations supporting the existence of correlated topological phases. The talk will conclude with an outlook on future opportunities and challenges in the study of twisted bilayer $MoTe_2$.