In recent years we have shown that relatively small systems can already give access to non-equilibrium steady state physics. A key aspect is to have local operators which relax quickly. In this work we investigate how local operators relax via the lens of ETH and hydrodynamics.

The eigenstate thermalization hypothesis (ETH) describes the properties of diagonal and off-diagonal matrix elements of local operators in the eigenenergy basis. We propose a relation between (i) the singular behaviour of the off-diagonal part of ETH at small energy differences, and (ii) the smooth profile of the diagonal part of ETH as a function of the energy density. We establish this connection from the decay of the autocorrelation functions of local operators, which is constrained by the presence of local conserved quantities whose evolution is described by hydrodynamics. We thus give a stronger basis to the overlap-relaxation inequality and we corroborate our predictions with numerical simulations of two non-integrable spin-1 Ising models, one diffusive and one super-diffusive, which we perform using dynamical quantum typicality up to 18 spins.